Instructions Answer all questions. Give your answers clearly. Do not skip intermediate steps even they are very easy. Complete answers with no error receive full mark; expressional and notational errors are not overlooked.

Calculators and cell phones are not allowed in the exam.

Time 120 minutes.

1	2	3	4	5	6	7	8	9	TOTAL

QUESTIONS

Q1) (20p) Evaluate the following mathematical operations. Compute only two values ($k = 0, 1$) of the operations in (a) and (e). Write your results in Cartesian form.

a) $\log\left(e^2\cos(3) + i\,e^2\sin(3)\right)$ b) $\cos\left(\pi/2 + i\,\text{Log}(6)\right)$

c) $\cosh\left(\text{Log}(4) - i\,\pi/2\right)$ d) $\sqrt[3]{8i}$ e) $(-e^{\pi})^{i}$.

Q2) (16 p) Determine and plot sets described by the following inequalities.

a) $-1 \leq \text{Re } z < 2$, b) $\pi > \text{Arg } z > \pi/4$, c) $|z + 1 - 2i| \leq 1$, d) $|z + 1 - 2i| > 1$.

Q3) (10 p) Find each of the following limits.

a) $\lim_{z \to -2i} \dfrac{z^3 + 4z}{z + 2i}$ b) $\lim_{z \to -1+i} \dfrac{2z^2 + 3z + 3 + i}{z^2 + 4z + 4 - 2i}$.

Q4) (15 p) Find $\dfrac{dw}{dz}$ for each of the following.

a) $w = \tanh z$ b) $w = i\sin\left(\dfrac{1}{z}\right) + \cos(2z)$ c) $w = [\sinh z + 1]^2$.

Q5) (12 p) Check analyticity of the following functions with Cauchy-Riemann equations. Remember that x and y are real and imaginary parts and r and θ are magnitude and phase of complex variable $z = x + iy = re^{j\theta}$.

a) $w(x, y) = 2x^2 - 2y^2 - 3x + i\,(4xy - 3y)$ b) $w(r, \theta) = 3r\cos(\theta) + i\,r\sin(\theta)$

c) $w(z) = z^2 + 2\bar{z}$.

Q6) (8 p) Consider the complex valued function $g(\theta) = e^{i\theta}, \quad \theta \in \mathbb{R}$. Since $g''(\theta) = -e^{i\theta}$, the following second order differential equation can be derived.

$$g''(\theta) + g(\theta) = 0$$

a) Is $g(\theta) = A\cos(\theta) + B\sin(\theta)$ also a solution of this differential equation? b) If your answer is yes, obtain the coefficients A and B that make $A\cos(\theta) + B\sin(\theta) = e^{i\theta}$.

Q7) (10 p) Compute the length of the curve traced by $z(t) = t + i\frac{1}{2}t^2, \quad 0 \le t \le 1$. When needed you can use the following mathematical expressions.

$$\int \sqrt{1+t^2}\,dt = \frac{\sinh^{-1}(t)}{2} + \frac{t\sqrt{t^2+1}}{2} + \text{constant}$$

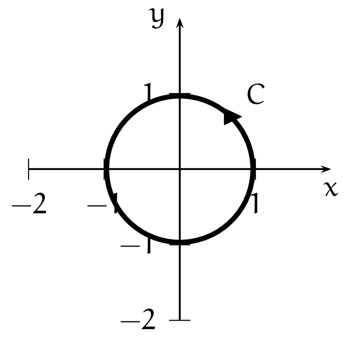
$$\sinh\left(\log\left(1+\sqrt{2}\right)\right) = 1, \qquad \log\left(1+\sqrt{2}\right) \approx 0.88$$

$$\sqrt{2} \approx 1.41$$

Q8) (19 p)

a) Compute the contour integral of $f(z) = z^n$ along $C : z(t) = \cos(t) + i\sin(t), \quad 0 \le t < 2\pi$, where n is an integer; $\oint_C f(z)\,dz$.

b) Compute the contour integral; $\oint_C f(z)\,dz$, of



Q8

$$f(z) = \frac{2\left(3z^3 + 10z^2 + 14z + 15\right)}{z(2z+3)\left(z^2+2z+5\right)} = \frac{2}{z} + \frac{1}{z+1-2i} + \frac{1}{z+1+2i} - \frac{1}{z+\frac{3}{2}}$$

over a closed contour C traversed once in the counter-clockwise direction enclosing the origin as indicated in the figure.

Q9) (10 p) Evaluate

$$\oint_C \left[\frac{2}{z-i} + \frac{6}{(z-i)^2} - 3\,(z-i)^2 + 1 \right] dz,$$

where C is a closed contour traversed once in the counter-clockwise direction enclosing $z = i$.