Digital Signal Processing

Lecture Notes and Exam Questions

Convolution Sum of Two Finite Sequences

Consider convolution of $h(n)$ and $g(n)$ $(M > N)$;

$$
\begin{aligned}
&h(n), \quad n = 0 \ldots M-1 \\
&g(n), \quad n = 0 \ldots N-1 \quad .
\end{aligned}
$$

$$
y(n) = \begin{cases}
\sum\limits_{k=0}^{n} h(k)\ g(n-k), & 0 \leq n \leq M-1 \\
\sum\limits_{k=n-M+1}^{n} h(k)\ g(n-k), & M \leq n \leq N-1 \\
\sum\limits_{k=n-M+1}^{N-1} h(k)\ g(n-k), & N \leq n \leq N+M-2 \\
0, & elsewhere \quad .
\end{cases}
$$

Find complex convolution, $V(z)$, of the following z-transforms.

$$X(z) = \frac{z}{z-1} \quad |z|\rangle\, 1 \quad , \qquad\qquad Y(z) = \frac{2z}{2z-1} \quad |z|\rangle\, \frac{1}{2} \quad .$$

Complex convolution is defined as

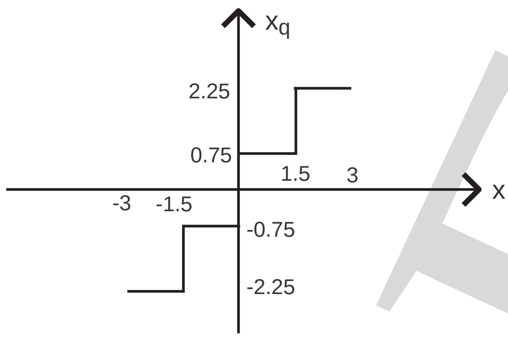
$$V(z) = \frac{1}{2\pi j}\oint_{\Gamma} X\left(\frac{z}{w}\right) Y(w)\, w^{-1} dw$$

where Γ is the integration contour and must be chosen within the intersection of region of convergence of $X\left(\frac{z}{w}\right)$ and $Y(w)$.

EEM 409 Digital Signal Processing Midterm Exam November 26, 2001

QUESTIONS

Q1.



quantizer levels	codes
-2.25	00
-0.75	01
0.75	10
2.25	11

n	0	1	2	3	4	5	6	7
$x(n)$	-2.1	-0.3	1.85	0.8	1	1.8	-1.2	-0.8

Find $x_q(n)$ and total number of bits to represent $x_q(n)$.

Q2.

n	0	1	2	3	4	5	6	7
$x(n)$	2	1	3	-1	2	1	3	2

Using FFT algorithm find Fourier Transform of the signal at 8 points.

Q3.

n	0	1	2	3	4	5	6	7
$x(n)$	1	2	2	1	1	2	2	1

Calculate Fourier Transform of the signal at 8 points and plot magnitude and phase of the transform.

Q4.

$$X(z) = \frac{4z}{(4z-1)^2}, \quad \text{ROC} = |x| > \frac{1}{4}$$

Find $x(n)$ using inverse z-Transform formula.

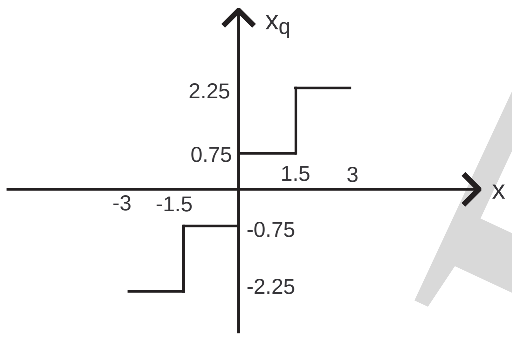
Good Luck

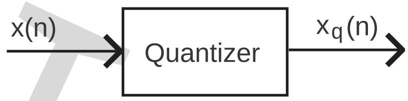
Sami Arıca

EEM 409 Digital Signal Processing Final Exam January 22, 2002

QUESTIONS

Q1.





quantizer levels	codes
-2.25	000
-0.75	001
0.75	01
2.25	1

n	0	1	2	3	4	5	6	7
$x(n)$	-2.1	-0.3	1.85	0.8	1	1.8	-1.2	-0.8

Find $x_q(n)$ and codes assigned to the samples. Find total number of bits to represent $x_q(n)$.

Q2.

n	0	1	2	3	4
$x(n)$	2	1	-1	1	-2

Find Discrete Fourier Transform of the signal at 6 points.

$X(6-k) = X^*(k)$

Q3. Calculate magnitude and phase of the following discrete Fourier transform.

$$X(k) = e^{-j2\pi k/3} \cdot \left(2\cos\frac{\pi}{3}k - 1 + j\,4\sin\frac{2\pi}{3}k\right), \quad k = 0, \ldots, 5$$

Q4.

$$X(z) = \frac{1}{1-0.5z^{-1}} + \frac{1}{1-0.5z}, \quad \text{ROC} = 0.5 < |z| < 2$$

Find $x(n)$, a) using inverse z-Transform formula, b) using partial fraction expansion method.

Good Luck

Sami Arıca

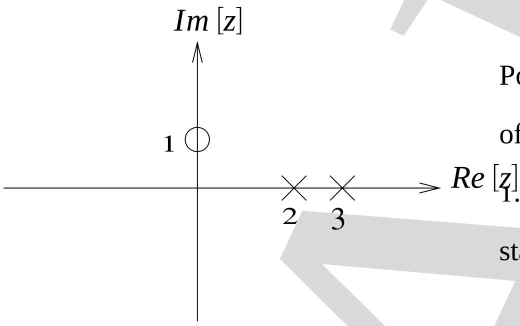
QUESTIONS

Q1) Frequency response of a linear shift invariant system is given as

$$H(\Omega) = j\frac{\sin(\Omega)}{1+\cos(\Omega)} \quad .$$

Find the difference equation of the system.

Q2)



Pole-zero plot of system response, $H(z)$, of a system is as in the figure. $H(0) = 1$. Find $H(z)$. What can you say about stability of the system ?

Q3) Let $X(\Omega)$ be the Fourier Transform of $x(n) = (1/2)^n u(n)$. Let $Y(k)$ be DFT with $N = 4$ samples of a finite-length signal $y(n)$. We set

$$Y(k) = X(\Omega)\,|_{\Omega=2\pi k/4} \qquad k = 0\ldots 3 \quad .$$

Determine $y(n)$.

Q4) Find z-transform of $x(n) = \left(\frac{1}{2}\right)^{|n|}$.

Good Luck.

EEM 409 Digital Signal Processing - December 19, 2003

QUESTIONS

Q1) Draw diagram of radix-2 FFT algorithm for 4 points. Find FFT of $x = [1, -2, 4, 2]$.

Q2) Using DFT find convolution of the following sequences.

$$x = [1, 2]$$
$$y = [2, -3]$$

Q3) Design half band low pass discrete-time filter using Fourier series method. Choose filter length as 7 and use hamming window. (Hamming window:

$$w(n) = \begin{cases} 0.54 + 0.46\cos\left(\frac{2\pi n}{N-1}\right), & |n| \leq \frac{N-1}{2} \\ 0 & otherwise \end{cases}$$

)

Q4) Find z-transform of $x(n) = \left(\frac{1}{2}\right)^n u(n)$. Evaluate the transform at points, $z_l = A \cdot W^{-l}, \qquad l = 0, \ldots .\ 9$ where $A = e^{j\pi/6}$ and $W = 0.995e^{-j\pi \cdot 0.25}$.

Good Luck

Sami Arıca

EEM 409 Digital Signal Processing - December 19, 2003

ANSWERS

A1) Consider that discrete Fourier transfor of sequence $g(n)$ is G_k.

$$G_k = \sum_{n=0}^{N/2-1} \left(g_n + g_{n+N/2} W^{kN/2} \right) W^{kn}, \quad k = 0, 1, \ldots, N-1$$

where $W = e^{-j2\pi/N}$ and $W^{N/2} = -1$. Let,

$$\begin{aligned} x_n &= g_n + g_{n+N/2} \\ y_n &= g_n - g_{n+N/2} \end{aligned}$$

Hence the transform coefficients can be witten as follows,

$$\begin{aligned} G_{2l} &= \sum_{n=0}^{N/2-1} x_n W_{N/2}^{ln}, \quad l = 0, 1, \ldots, N/2 - 1 \\ G_{2l+1} &= \sum_{n=0}^{N/2-1} \left(y_n W^n \right) W_{N/2}^{ln} \end{aligned}$$

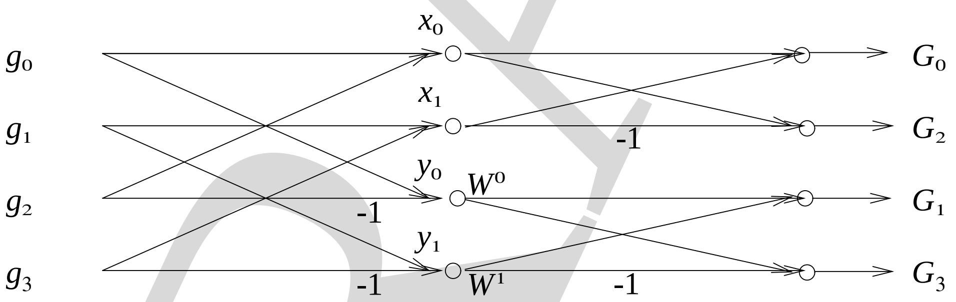


Figure 1: Block diagram of four point FFT

Here $N = 4$ and $W = e^{-j2\pi/4} = -j$. $g_0 = 1$, $g_1 = -2$, $g_2 = 4$, and $g_3 = 2$. $x_0 = 5$, $x_1 = 0$, $y_0 = -3$, and $y_1 = -4$.

$$\begin{aligned} G_0 &= x_0 + x_1 &&= 5 \\ G_2 &= x_0 - x_1 &&= 5 \\ G_1 &= y_0 + W \cdot y_1 &&= -3 + j4 \\ G_3 &= y_0 - W \cdot y_1 &&= -3 - j4 \end{aligned}$$

A2) Length of $x(n)$ and $x(n)$ are $N = 2$ and $M = 2$ respectively. $a(n) = x(n) \star y(n)$.

Length of $a(n)$ is $N + M - 1 = 3$.

$$
\begin{aligned}
X(k) &= \sum_{n=0}^{1} x(n)\, e^{-j(2\pi/3)nk}, \qquad k = 0, 1, 2. \\
Y(k) &= \sum_{n=0}^{1} y(n)\, e^{-j(2\pi/3)nk}, \qquad k = 0, 1, 2. \\
A(k) &= X(k) \cdot Y(k), \qquad k = 0, 1, 2. \\
a(n) &= \frac{1}{3} \sum_{k=0}^{2} A(k)\, e^{j(2\pi/3)nk}, \qquad n = 0, 1, 2.
\end{aligned}
$$

$$
\begin{aligned}
X(k) &= 1 + 2e^{-j(2\pi/3)k} \\
Y(k) &= 2 - 3e^{-j(2\pi/3)k}
\end{aligned}
$$

$$
\begin{array}{lll}
X(0) = 3 & X(1) = 1 + 2e^{-j2\pi/3} = -1.7321j & X(2) = 1 + 2e^{-j4\pi/3} = 1.7321j \\
Y(0) = -1 & Y(1) = 2 - 3e^{-j2\pi/3} = 2.5 + 2.5981j & Y(2) = 1 + 2e^{-j4\pi/3} = 2.5 - 2.5981j \\
A(0) = -3 & A(1) = 4.5 - 6.0622j & A(2) = 4.5 + 6.0622j
\end{array}
$$

Inverse DFT of $A(k)$ gives,

$$
IDFT(A(k)) = a(n) = \begin{cases} 2, & n = 0 \\ 1, & n = 1 \\ -6, & n = 2 \\ 0, & \textit{otherwise} \end{cases}
$$

A3)

$$H(\Omega) = \begin{cases} 1, & |\Omega| \leq \pi/2 \\ 0, & \pi/2 < |\Omega| < \pi \\ H(\Omega + 2\pi \cdot l), & l \in Z \end{cases}$$

EEM 409 Digital Signal Processing Final Exam - January 19, 2004

QUESTIONS

Q1) Derive in frequency FFT algorithm and draw block diagram of 4 points. Find FFT of $x = [-1, 2, 4, -3]$ using the algorithm.

Q2) The response of an LTI system to input $x(n)$ is $y(n)$. DFT of the input and output signals are $X = [-2, 3+5i, 8, 3-5i]$ and $Y = [-12, 4-16i, 16, 4+16i]$ respectively. Find impulse response $h(n)$ of the system.

Q3) The characteristics of an ideal high pass filter is given as,

$$H(\Omega) = \begin{cases} 0, & 0 \le |\Omega| < 2\pi/3 \\ 1, & 2\pi/3 \le |\Omega| < \pi \end{cases}$$

. Design this filter using Fourier series method. Choose filter length as 7 and use hamming window. (Hamming window:

$$w(n) = \begin{cases} 0.54 + 0.46\cos\left(\frac{2\pi n}{N-1}\right), & |n| \le \frac{N-1}{2} \\ 0 & otherwise \end{cases}$$

)

Q4) Find z-transform of $x(n) = \left(\frac{1}{2}\right)^n u(n)$. Evaluate the transform at points, $z_l = A \cdot W^{-l}, \quad l = 0, 1, 2$ where $A = e^{j\pi/6}$ and $W = 0.5e^{-j\pi \cdot 0.25}$.

Q5) Impulse response of a continuous time system is given as,

$$h(t) = \begin{cases} \cos(\pi \cdot t/4), & |t| \le 4 \\ 0, & otherwise \end{cases}$$

. Employ sampling period of $T = 0.25sec$. Obtain impulse response of discrete time approximation of the system by using impulse invariance method.

Q6) System function of an LTI system can be written as,

$$H_A(s) = \sum_{k=1}^{N} \frac{A_k}{s - p_k}$$

. Find impulse response $h_A(t)$ of the system. Using impulse invariance method, find impulse response, $h_D(n)$, and system function, $H_D(z)$ of discrete time approximation of the analog system. The sampling period is T.

Good Luck

Sami Arıca

QUESTIONS

Q1) Find state-space representation for an LTI system given as,

$$y(n) - 2y(n-1) + y(n-2) = x(n)\ .$$

Q2) Transfer function of an LTI system is given as,

$$H(z) = \frac{-2z^2 + 16z - 12}{z^3 - 5z^2 + 6z}\ .$$

Find impulse response of the system by using partial fraction expansion method.

Q3) Find 3×3 DFT matrix and calculate DFT of $x(n) = 3\delta(n) + \delta(n-1) + 2\delta(n-2)$ by using the matrix representation of DFT.

Q4) Design a low-pass filter using Fourier Series method. The bandwidth of the filter is $\pi/2$ and the length is $N = 5$. Employ Blackman window,

$$w(n) = 0.42 + 0.5\cos\left(\frac{2\pi n}{N-1}\right) + 0.08\cos\left(\frac{4\pi n}{N-1}\right), \quad n = -\frac{N-1}{2}, \cdots, \frac{N-1}{2}\ ,$$

to reduce ringing effect.

QUESTIONS

Q1) Write Decimation in Time (DIT) Radix-2 FFT algorithm.

Q2) Design half band low pass filter by using 2rd order butterworth approximation.

Q3) Find Fourier transform and energy spectrum of $x(n) = -1/\sqrt{6}\,\delta(n) + 2/\sqrt{6}\,\delta(n-1) - 1/\sqrt{6}\,\delta(n-2)$.

Q4) $x(n) = \cos(0.1\pi \cdot n/3) + \cos(0.1\pi n)$ is given. This signal is an input to an LTI filter with impulse response $h(n) = a\delta(n) + b\delta(n-1) + a\delta(n-2)$. The output of the filter is $y(n) = \cos(0.1\pi n)$. Find filter coefficients.

Q5) Sampling frequency of a discrete signal is increased twice by digital interpolation; $y(n) = xup(n) * h(n)$, where

$$xup = \begin{cases} x(n/2), & n = 0, \mp 2, \mp 4, \ldots \\ 0, & otherwise \end{cases}$$

and $h(n)$ is a half band digital filter.

Let $x(t) = \cos(2\pi t)$. Sample this signal with sampling frequency $f_s = 5Hz$ (sampling period $T_s = 1/f_s = 0.2\ sec.$) and obtain $x(n)$. Using the digital interpolation method and $h(n) = 1/\sqrt{2}\,\delta(n) + 1/\sqrt{2}\,\delta(n-1)$ find interpolated signal $y(n)$. Calculate $y(n)$ for $n = 0.1(2k+1)\ \ sec,$ where, $k = 0, 1, 2, 3, 4$.

Digital Signal Processing Homework

1. MATLAB implementation of Decimation in Time (DIT) Radix-2 FFT (see http://cnx.rice.edu/content/m12016/latest).

2. Design half band low pass filter

   (a) by using Fourier series method (length N = 11). Employ Blackman window.

   (b) by using 3rd order butterworth approximation (see http://www-sigproc.eng.cam.ac.uk/ ad2/3f3/3F3-3.doc and http://www.stanford.edu/class/ee102b/lectures/lecnotes13.pdf).

3. Find Fourier transform ($X(\omega)$ and energy spectrum ($X(\omega) X^{*}(\omega)$) of audio signal http://www.members.tripod.com/ buggerluggs/wavs/hello.wav. Calculate Fourier transform at 8192 points.

4. Output of an LTI system with frequency response of $H(\omega)$ for an input signal $x(n) = \cos(\omega_a n) + \cos(\omega_b n)$ is $y(n) = H(\omega_a)\cos(\omega_a n) + H(\omega_b)\cos(\omega_b n)$. Let $h(n) = \mathcal{F}^{(}-1)[H(\omega)] = a\delta(n) + b\delta(n-1) + a\delta(n-2)$. Find $h(n)$ such that $y(n) = h(n) * x(n) = \cos(\omega_a n)$. $\omega_a = 0.1\pi/3$ and $\omega_b = 0.1\pi$.

5. Sampling frequency of a discrete signal is increased twice by digital interpolation; $y(n) = xup(n) * h(n)$, where

$$xup = \begin{cases} x(n/2), & n = 0, \mp 2, \mp 4, \ldots \\ 0, & otherwise \end{cases}$$

and $h(n)$ is a half band digital filter.

Let $x(t) = \cos(2\pi t)$. Sample this signal with sampling frequency $f_s = 5Hz$ (sampling period $T_s = 1/f_s = 0.2sec.$) and obtain $x(n)$. Using the digital interpolation method and one of the halfband filters designed in Q2, increase the sampling frequency of $x(n)$ to $10Hz$.

6. Let $x(n) = x_a(n) + x_b(n)$. $x_a(n) = \cos(0.1\pi n)$ and $x_b(n) = 2\sin(0.01\pi n)$. Here $x_b(n)$ is unwanted signal which shift the base of $x_a(n)$, therefore it is called trend of $x(n)$. We want to remove the trend. Use the linear trend removal algorithm:

   - segment $x(n)$ into small parts, $x(n) = \sum_k x_k(n - kN)$, where $x_k(n) = x(n + kN)$, $n = 0, \ldots N - 1$.
   - for k th segment find linear trend ; $an + b$,
   - remove the linear trend from the k th segment; $y_k(n) = x_k(n) - an - b$, $n = 0 \ldots N - 1$.
   - $y(n) = \sum_k y_k(n - kN)$ is the output of the algorithm. $y(n) \approx x_a(n)$.

   For this example choose $N = 5$ and remove the trend of $x(n)$ for $n = 0 \ldots 299$.

EE597 Discrete Signal and System Analysis Midterm Exam December 20, 2001

QUESTIONS

A linear time-invariant and causal system which is described by

$y(n) - (a+b) \cdot y(n-1) + ab \cdot y(n-2) = c \cdot x(n-2)$

where $|a| < 1$ and $|b| < 1$ is given.

Q1. Find the impulse response of the system.

Q2. Find the frequency response of the system.

Q3. Plot block diagram and flow graph of the system.

Q4. Obtain state-space characterization of the system.

EE597 Discrete Signal and System Analysis Midterm Exam January 30, 2002

QUESTIONS

A linear time-invariant and causal system which is described by
$y(n) - 2a \cdot y(n-1) + a^2 \cdot y(n-2) = a \cdot x(n-1)$ where $a = 0.5$, is given.

Q1. Find the impulse response of the system using time-domain analysis.

Q2. Find the the frequency response of the system.

Q3. Plot block diagram and flow graph of the system.

Q4. Obtain state-space characterization of the system.

Q5. Transfer function of a linear time-invariant system is given as following. Find input-output relation in time domain. Check if the system is causal and stable.

$$H(z) = \frac{2-z}{z^{-2} - 0.25}$$

QUESTIONS

Q1) Design a low-pass filter using Fourier Series method. The bandwidth of the filter is $\pi/3$ and the length is $N = 5$. Employ Hamming window,

$$w(n) = 0.54 + 0.46\cos\left(\frac{2\pi n}{N-1}\right), \quad n = -\frac{N-1}{2}, \cdots, \frac{N-1}{2},$$

to reduce ringing effect.

Q2) A sequence of symbols are given ; $\{a, b, b, c, a, c, b, c, c, b\}$. The following codes are assigned to the symbols.

$$\begin{array}{ll} a, & 00 \\ b, & 01 \\ c, & 1 \end{array}$$

a) Find probability of symbols a, b, c. b) Calculate information of the symbols and entropy of the sequence. c) What is the average code length (bits/symbol)?

Q3) Find inverse z-transform of the following z-transform using the inverse z-transform formula.

$$X(z) = \frac{z}{(z-1)^2}, \quad |z| > 1$$

Recall :

$$\begin{aligned} \frac{1}{2\pi j}\oint_{\Gamma} F(z)\,dz &= \sum_k \left(Res_q^{\alpha}\right)_k \\ Res_q^{\alpha} &= \frac{1}{(\alpha-1)!}\lim_{z\to q}\frac{d^{\alpha-1}}{dz^{\alpha-1}}(z-q)^{\alpha}F(z) \end{aligned}$$

where $z = q$ is α multiple pole of $F(z)$ and $\left(Res_q^{\alpha}\right)_k$ is the k th resudue.

Q4) Numeric approximation of integral of $x(t)$ is,

$$\begin{aligned} y(n\Delta) &= \sum_{k=-\infty}^{n} \frac{x((k-1)\Delta) + x(k\Delta)}{2}\Delta \\ &= y((n-1)\Delta) + \frac{x((n-1)\Delta) + x(n\Delta)}{2}\Delta \end{aligned}$$

Since $y(n\Delta) \to y(n)$ and $x(n\Delta) \to x(n)$,

$$y(n) = y(n-1) + \frac{x(n-1) + x(n)}{2}\Delta$$

This equation can be considered as a discrete-time system. Find a) the system response, b) the frequency response of the system.

ANSWERS

A1)

$$\begin{aligned} g(n) &= \frac{1}{2\pi}\int_{-\pi/3}^{0} e^{j\omega n} dw + \frac{1}{2\pi}\int_{0}^{\pi/3} e^{j\omega n} dw \\ &= \frac{1}{\pi}\int_{0}^{\pi/3} \cos(\omega n)\, dw \\ &= \frac{1}{\pi n}\sin(\pi n/3) \end{aligned}$$

$$\begin{aligned} h(n) &= w(n) \cdot g(n) \\ h(n) &= h(-n) \end{aligned}$$

$$\begin{aligned} h(0) &= 0.3333 \\ h(1) &= 0.1489 \\ h(2) &= 0.0110 \end{aligned}$$

A2)

$$\begin{aligned} p(a) &= \frac{2}{10} \\ p(b) &= \frac{4}{10} \\ p(c) &= \frac{4}{10} \end{aligned}$$

$$\begin{aligned} I(a) &= -\log_2(p(a)) \\ &= 2.3219 \end{aligned}$$

$$\begin{aligned} I(b) &= -\log_2(p(b)) \\ &= 1.3219 \end{aligned}$$

$$\begin{aligned} I(c) &= -\log_2(p(c)) \\ &= 1.3219 \end{aligned}$$

$$\begin{aligned} H &= p(a)\,I(a) + p(b)\,I(b) + p(c)\,I(c) \\ &= 1.5219 \;\; bits \end{aligned}$$

Coded sequence : 00 01 01 1 00 1 01 1 1 01

$$A = \frac{6\cdot 2 + 4\cdot 1}{10} = \frac{16}{10} = 1.6$$

A3)

$$\begin{aligned} x(n) &= \frac{1}{2\pi j}\oint_{\Gamma} X(z)\,z^{n-1}dz \\ &= \frac{1}{2\pi j}\oint_{\Gamma} \frac{z^n}{(z-1)^2}\,dz \end{aligned}$$

$$F(z) = \frac{z^n}{(z-1)^2}$$

For $n \geq 0$, $z = 1$ is pole of $F(z)$ and $q = 1$ and $\alpha = 2$.

$$Res_1^2 = \frac{1}{1!}\lim_{z\to 1}\frac{d}{dz}(z-1)^2 F(z) = \lim_{z\to 1} nz^{n-1} = n$$

For $n = -m$ and $m > 0$ there is two poles at $z = 0$ and at $z = 1$. Consider pole at $z = 0$,

$$F(z) = \frac{1}{z^m (z-1)^2}$$

$$\frac{d^{m-1}}{dz^{m-1}}\left(z^m F(z)\right) = m!\;\frac{(-1)^{m+1}}{(z-1)^{m+1}}$$

$$Res_0^m = \frac{1}{(m-1)!}\lim_{z\to 0}\frac{d^{m-1}}{dz^{m-1}}\left(z^m F(z)\right) = \lim_{z\to 0}\left(m\;\frac{(-1)^{m+1}}{(z-1)^{m+1}}\right) = m$$

Consider the pole at $z = 1$,

$$Res_1^2 = \frac{1}{1!}\lim_{z\to 1}\frac{d}{dz}(z-1)^2 F(z) = \lim_{z\to 1}\frac{-m}{z^{m+1}} = -m$$

$$Res_0^m + Res_1^2 = 0$$

Then the inverse transform for any n is,

$$x(n) = nu(n)$$

A4)

$$Y(z) = z^{-1}\,Y(z) + \frac{z^{-1}\,X(z) + X(z)}{2}\,\Delta$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\Delta}{2}\cdot\frac{1+z^{-1}}{1-z^{-1}}$$

$$\begin{aligned} H(\omega) &= H(z)\Big|_{z=e^{j\omega}} \\ &= \frac{\Delta}{2}\cdot\frac{1+e^{-j\omega}}{1-e^{-j\omega}} \\ &= \frac{\Delta}{2}\cdot\frac{e^{-j\omega/2}2\cos(\omega)}{e^{-j\omega/2}2j\sin(\omega)} \\ &= \frac{\Delta}{2}(-j)\cot(\omega) \end{aligned}$$

For continuous case,

$$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$$

and the system function is

$$H(s) = \frac{1}{s}$$

Equating the system functions of discrete and continuous systems,

$$\frac{1}{s} = \frac{\Delta}{2} \cdot \frac{1 + z^{-1}}{1 - z^{-1}}$$

results with

$$s = \frac{2}{\Delta} \cdot \frac{1 - z^{-1}}{1 + z^{-1}}$$

Using this relation it is possible to transform a continuous time system to a discrete time system. This transformation is called as bilinear transform.

QUESTIONS

Q1) Design a high-pass filter using Fourier Series method. The passband of the filter is $[\pi/2,\ \pi]$ and the length is $N = 5$. Employ Hamming window,

$$w(n) = 0.54 + 0.46\cos\left(\frac{2\pi n}{N-1}\right), \quad n = -\frac{N-1}{2}, \cdots, \frac{N-1}{2},$$

to reduce ringing effect.

Q2) A sequence of symbols are given ; $\{0,\ 1,\ 2,\ 2,\ 2,\ 2,\ 2,\ 3,\ 3,\ 3,\ 3,\ 0,\ 0,\ 2,\ 2\}$. The following codes are assigned to the symbols.

$$\begin{array}{ll} 0, & 000 \\ 1, & 001 \\ 2, & 1 \\ 3, & 01 \end{array}$$

a) Find probability of symbols $0,\ 1,\ 2,\ 3$. b) Calculate information of the symbols and entropy of the sequence. c) What is the average code length (bits/symbol)?

Q3) Find inverse z-transform of the following z-transform using the partial fraction expansion method.

$$X(z) = -2\,\frac{z(4z-3)}{(2z-1)(4z-1)}, \quad |z| > \frac{1}{2}$$

Q4) *n th* order analog Butterworth low pass filter is defined as,

$$H(s)\,H(-s) = \frac{1}{1 + (-1)^n \left(\frac{s}{\omega_a}\right)^{2n}} \quad .$$

where $H(s)$ is the system function and ω_a is the bandwith of the filter in rad/sec. First order Butterworth filter is,

$$H(s)\,H(-s) = \frac{1}{1-\left(\frac{s}{\omega_a}\right)^2} = \frac{1}{1+\frac{s}{\omega_a}}\,\frac{1}{1-\frac{s}{\omega_a}} \quad .$$

The system function is,

$$H(s) = \frac{1}{1+\frac{s}{\omega_a}}$$

since the pole $s = \omega_a$ resides in the left side of the s-plane. Using bilinear transform;

$$s = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}}$$

find discrete approximation of the first order analog Butterwort filter for the bandwith $\omega_a = 2\pi\ rad/sec$ and sampling interval $T = 2\ sec$. Employ pre-warping;

$$\omega_d = \frac{2}{T}\arctan\left(\frac{\omega_a\,T}{2}\right) \quad ,$$

where ω_d is discrete equivalent of analog frequency ω_a in rad/sec. To avoid warping ω_a should be replaced by $\frac{2}{T}\arctan\left(\frac{\omega_a\,T}{2}\right)$ in $H(s)$. First find discrete equivalent $H(z)$ of $H(s)$ using the bilinear transform and then write difference equation of the discrete filter.