QUESTIONS

Q1. Find frequency response and plot magnitude and phase spectra of the LTI-casual system

$$y(n) = x(n) + 2x(n-2) + x(n-4)$$

Q2. One period of a periodic signal is given in the following. Find Fourier series coefficients of this signal.

n	0	1	2	3	4	5
x(n)	3	2	1	-1	-2	-3

Q3. A continuous-time signal x (t) = $e^{-t}u(t)$ is sampled at a sampling rate $f_s = 4$ Hz. Obtain its samples, x (n/f_s), for n = 0, 1, 2, ..., 7. Assume that the samples are zero for n > 7. Compute x ((n + 1/2) / f_s) for n = 0, 1, 2, ..., 7 using the function (continuous-time signal) itself and the sinc interpolation formula

$$x(t) = \sum_{n=-\infty}^{\infty} x(n) \operatorname{sinc} (f_{s}t - n)$$

Q4. Construct a 4-bit (R = 4) uniform quantizer for the input range [-A, A] (A > 0). Note that

$$\begin{split} L &= 2^{R}, \qquad \Delta = \frac{2A}{L}, \qquad A = \frac{L}{2}\Delta \\ \xi_{k} &= \left[\left(k - L/2 \right) \Delta, \left(k + 1 - L/2 \right) \Delta \right), \qquad k = 0, 1, 2, \dots, L-1 \\ q_{k} &= \left(k + 1/2 \right) \Delta - L\Delta/2 \end{split}$$

Q5. Consider an LTI-casual system with an impulse response $h(n) = a^n u(n)$, and $a \in \mathbb{C}$. a) Show that the system is stable if |a| < 1. b) Demonstrate that the system is described by following first order difference equation.

$$\mathbf{y}\left(\mathbf{n}\right) - \mathbf{a}\mathbf{y}\left(\mathbf{n}-\right) = \mathbf{x}\left(\mathbf{n}\right)$$

c) Suppose that the following first order systems are cascaded (connected in series).

$$w(n) - aw(n-1) = x(n)$$
 $x(n)$ is the input, $w(n)$ is the output
 $y(n) - by(n-1) = w(n)$ $w(n)$ is the input, $y(n)$ is the output $(b \neq a)$

Show that this cascaded system is equivalent to the following second order LTI-causal system

$$y(n) - (a + b) y(n - 1) + aby(n - 2) = x(n)$$

d) Find impulse response of this system.

e) Prove that above second order system is stable if roots of its characteristic equation are inside the unit circle of the complex plane.