Instructions Answer all questions. Give your answers clearly. Each question is worth 15 points. **Time** 90 minutes.
Good Luck.

QUESTIONS

Q1. Find the impulse response of the system of the following LTI-causal system (solve the difference equation in time domain).

$$y(n) - y(n-1) + \frac{1}{4}y(n-2) = x(n) + 4x(n-1) - \frac{13}{4}x(n-2) + \frac{3}{4}x(n-3)$$

....

Hint:

$$y(n) = \left[\frac{2}{\left(1 - \frac{1}{2}\mathcal{D}^{-1}\right)^2} - 1 + 3\mathcal{D}^{-1} \right] x(n)$$

Q2. z-transform of a signal is given as

$$X(z) = \frac{-4 + \frac{7}{6}z^{-1} + \frac{1}{6}z^{-2}}{1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}}$$

....

Hint: Obtain partial fraction expansion of $X(z)$, and then use table of z-transforms to write the inverse transform.

$$X(z) = \frac{A}{1 - az^{-1}} + \frac{B}{1 - bz^{-1}} + C$$

Find the signal, $x(n)$ by computing inverse z-transform.

Some z-Transforms

$$\delta(n-k) \quad \overset{\mathcal{Z}}{\longleftrightarrow} \quad z^{-k}$$

$$a^n u(n) \quad \overset{\mathcal{Z}}{\longleftrightarrow} \quad \frac{1}{1 - az^{-1}} = 1 + \frac{az^{-1}}{1 - az^{-1}} \qquad |a| < 1$$

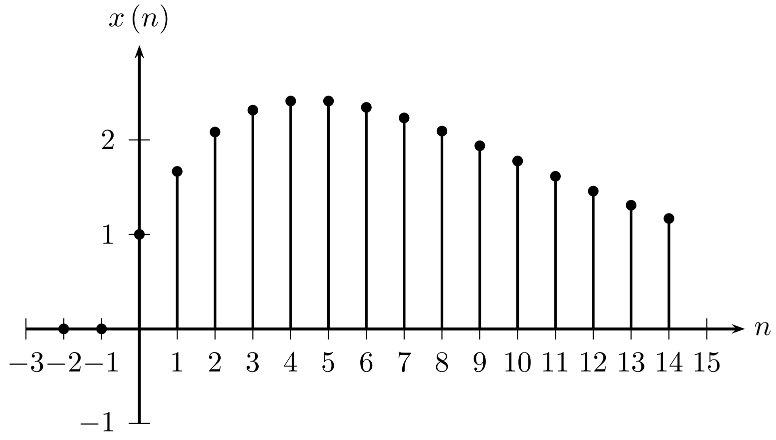
$$(n+1)\,a^n u(n) \quad \overset{\mathcal{Z}}{\longleftrightarrow} \quad \frac{1}{(1 - az^{-1})^2} \qquad |a| < 1$$

Q3. a) Compute Fourier transform of $\delta(n)$ and write the inverse Fourier transform to find a representation of the impulse. b) Compute Fourier series coefficients of $\sum_{\ell=-\infty}^{\infty} \delta(n - N\ell)$

and write the Fourier series of the impulse train.

Q4. Find the difference; $y(n) = x(n) - x(n-1)$, of the signal; $x(n) = (n+1)\left(\frac{5}{6}\right)^n u(n)$. Using the following algorithm compute the maximum value of the signal.

If $y(n) > 0$ and $y(n+1) < 0$, then maximum is $x(n)$



Q5. Find sum; $y(n) = \sum_{k=-\infty}^{n} x(k)$, of the signal; $x(n) = a^n\left(1 - \frac{1}{a}\right)u(n) + \frac{1}{a}\delta(n)$.

Q6. Design a high-pass half-band FIR ($\Omega_0 = \frac{\pi}{2}$) filter of order 6 ($2N+1 = 7$) by Fourier series method. Employ Hamming window for windowing.

Hamming window

$$w(n) = \begin{cases} 0.54 + 0.46\cos\left(\frac{\pi}{N}n\right), & -N \le n \le N \\ 0, & \text{otherwise} \end{cases}$$

Q7. Consider three periodic signals. One period of each of the three periodic signals is given as in the following. Compute discrete Fourier series coefficients of each signal, in terms of a and b.

n	0	1	2	3
$x_1(n)$	a	b	$-b$	a
$x_2(n)$	a	b	b	a
$x_3(n)$	a	b	$-b$	$-a$