Instructions Answer all questions. Give your answers clearly. Each question is worth 17 points. **Time** 90 minutes.

QUESTIONS

Q1. Find the impulse response of the system of the following LTI-causal system.

$$y(n) - \frac{3}{4}y(n-1) = 5x(n) - \frac{33}{20}x(n-1) + \frac{9}{10}x(n-2)$$

Hint: The impulse response is in the form; $h(n) = A\alpha^n u(n) + B\delta(n) + C\delta(n-1)$.

Q2. Fourier transform of a signal is given as

$$X(\Omega) = -\frac{1 - \frac{1}{2}e^{-j\Omega}}{1 - \frac{1}{4}e^{-j\Omega}}$$

For $z = e^{j\Omega}$ it becomes

$$X(z) = -\frac{1 - \frac{1}{2}z^{-1}}{1 - \frac{1}{4}z^{-1}}$$

Find the signal, $x(n)$ by computing inverse Fourier transform.

Some Transforms

$$\delta(n-k) \overset{\mathcal{F}}{\longleftrightarrow} e^{-j\Omega k} \rightarrow z^{-k}$$

$$a^n u(n) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{1 - ae^{-j\Omega k}} \rightarrow \frac{1}{1 - az^{-1}} = 1 + \frac{az^{-1}}{1 - az^{-1}}$$

$$(n+1)a^n u(n) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{(1 - ae^{-j\Omega k})^2} \rightarrow \frac{1}{(1 - az^{-1})^2}$$

Q3. Find Fourier transform of the following signals.

a) $(n+1)a^n u(n)$, b) $(n+1)(n+2)a^n u(n)$.

Sum of some series

$$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}$$

$$\sum_{n=0}^{\infty} (n+1)\alpha^n = \frac{1}{(1-\alpha)^2}$$

$$\sum_{n=0}^{\infty} (n+1)(n+2)\alpha^n = \frac{2}{(1-\alpha)^3}$$

Q4. The following system is given. The system make difference of the input signal.

$$y(n) = x(n) - x(n-1)$$

An input is given as below. a) Compute the output for $n = 0, 1, 2, 3, 4$. Assume that $x(-1) = x(0)$.

n	0	1	2	3	4
$x(n)$	1	3	4	3	1

b) Using the following algorithm compute the maximum value of the signal.

If $y(n) > 0$ and $y(n+1) < 0$, then maximum is $x(n)$

Q5. The following system is given.

$$y(n) = x(n) + x(n-1)$$

An input is given as below. Compute the output for $n = 0, 1, 2, 3, 4, 5, 6, 7, 8$. Assume that $x(-1) = 0$.

n	0	1	2	3	4	5	6	7	8
$x(n)$	1	0	3	0	4	0	3	0	1

Q6. Design a low-pass half-band FIR ($\Omega_0 = \frac{\pi}{2}$) filter of order 6 ($2N + 1 = 7$) by Fourier series method. Employ Hamming window for windowing.

Hamming window

$$w(n) = \begin{cases} 0.54 + 0.46\cos\left(\frac{\pi}{N}n\right), & -N \leq n \leq N \\ 0, & \text{otherwise} \end{cases}$$