QUESTIONS

Q1) An LTI-causal system is given as in the following. Write the characteristic equation of the system. From the characteristic equation determine weather the system is stable or not . Find the impulse response of the system.

$$y(n) - y(n-1) + \frac{2}{9} y(n-2) = 12x(n) - 7x(n-1) + \frac{4}{3} x(n-2)$$

Hint:

$$h(n) = A(\alpha_1)^n u(n) + B(\alpha_2)^n u(n) + C\delta(n)$$

where α_1 and α_2 are roots of the characteristic equation.

Q2) An LTI-causal system is given as in the following. Write the characteristic equation of the system. From the characteristic equation determine whether the system is stable or not . Find the impulse response of the system.

$$y(n) - \frac{5}{6} y(n-1) + \frac{1}{6} y(n-2) = x(n) - \frac{1}{6} x(n-2)$$

Q3) Compute Fourier transformation of the following signals.
a) $a^n u(n)$ b) $(n+1) a^n u(n)$ c) $\delta(n-k)$ d) $e^{j\Omega_0 n}$ e) $2\cos(\Omega_0 n)$

Q4) Compute inverse Fourier transformation of the following transforms.
a) $\dfrac{1}{1-ae^{-j\Omega}}$ b) $\dfrac{(A+B)-(aB+bA)e^{-\Omega}}{1-(a+b)e^{-\Omega}+abe^{-2\Omega}} = \dfrac{A}{1-ae^{-j\Omega}} + \dfrac{B}{1-be^{-j\Omega}}$ c) $e^{-j\Omega k}$
d) $2\pi \sum\limits_{k=-\infty}^{\infty} \delta(\Omega - \Omega_0 - 2\pi k)$ e) $\dfrac{1}{(1-ae^{-j\Omega})^2}$

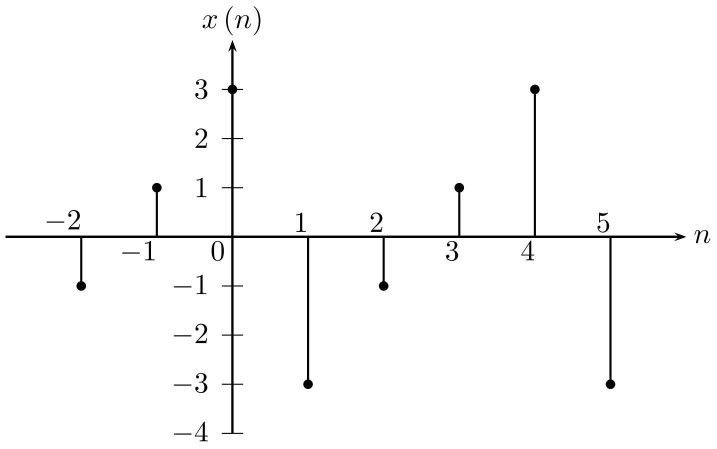
Q5) Prove the following Fourier transform properties.
a) $\mathcal{F}[nx(n)] = j\dfrac{d}{d\Omega} X(\Omega)$. Hint: Differentiate $X(\Omega)$.
b) $\mathcal{F}[x(n)\, y(n)] = \dfrac{1}{2\pi} \displaystyle\int_{-\pi}^{\pi} X(\Omega_1) Y(\Omega - \Omega_1)\, d\Omega_1$. Hint: In the right hand side of the equation write inverse Fourier transform representation of $Y(\Omega - \Omega_1)$.
c) $\mathcal{F}[x(n) * y(n)] = X(\Omega) Y(\Omega)$. Hint: In the left hand side of the equation write Fourier transform representation of $y(n)$ and then employ linearity property of the convolution.

Q6) Obtain the frequency response of the system given in Q1. Find the frequency response by computing inverse Fourier transformation of the frequency response.

Q7) Obtain the frequency response of the system given in Q1. Find the frequency response by computing inverse Fourier transformation of the frequency response.

Q8) Using Fourier series method design a half band low-pass FIR filter with an order 5. Employ Hamming window for the windowing.

Q9) Using Fourier series method design a half band high-pass FIR filter with an order 5. Employ Hamming window for the windowing.



Q10) Find Fourier series coefficients of the following periodic signal.

Q11) Complex Fourier series coefficients of a signal are given as in the following. Find the average power of the signal. Compute the inverse Fourier series to find the signal.

k	0	1	2	3
a_k	2	$2+2i$	-2	$2-2i$

Q12) Prove the following properties of complex Fourier series. $\mathcal{FS}\left[\,x(n)\,\right] = a_k$ and $\mathcal{FS}\left[\,y(n)\,\right] = b_k$. N is the common period of the signals.

a) $\mathcal{FS}\left[\,x(n)\;y(n)\,\right] = \sum_{\ell=0}^{N-1} a_\ell b_{k-\ell}$. Hint: In the right hand side of the equation write the analysis equation for $b_{k-\ell}$.

b) $\mathcal{FS}\left[\,x(n) * y(n)\,\right] = N a_k b_k$. Hint: In the left hand side of the equation write Fourier series representation of $y(n)$ and then employ linearity property of the convolution.

c) $\mathcal{FS}\left[\,a_n\,\right] = \frac{1}{N} x(-k)$.

d) $\mathcal{FS}\left[\sum_{n=0}^{N-1} x(n)\;y(n)\right] = \frac{1}{N}\sum_{k=0}^{N-1} a_k\, b_{-k}$.