QUESTIONS

Q1) Find inverse z-transform of the transfer function of an LTI-causal system given in the following.

$$H(z) = \frac{\left(2z^{-1}-1\right)\left(9z^{-2}+1\right)}{\left(1-\frac{1}{2}z^{-1}\right)^2\left(1+\frac{1}{4}z^{-2}\right)}$$

Q2) Derivative of a function at t = nT can be approximated as

$$\frac{dx(t)}{dt}\bigg|_{t=nT} \approx \frac{x(nT+T) - x(nT-T)}{2T}.$$

Using this approximation obtain a relation for a transformation from s-domain to z-domain. For the transformation you specify obtain the relation between analog frequency ω and digital frequency Ω .



Q2

Q3) Transfer function of a first order, causal analog low pass filter is

$$H(s) = \frac{1}{1+\frac{s}{a}} \qquad a > 0.$$

a) Using the transformation $s = \frac{1-z^{-1}}{T}$, obtain transfer function of corresponding digital filter. b) The pole of H(s) is s = -a and is in the left half plane of the complex plane. The pole of H(z) can be computed by $\frac{1-z^{-1}}{T} = -a$. Find the pole of H(z). Is it inside the unit circle?

Q4) Transfer function of a first order, causal analog low pass filter is

$$H(s) = \frac{1}{1+\frac{s}{a}} \qquad a > 0.$$

a) Using the bilinear transformation $s = \frac{2}{T} \frac{1-z^{-1}}{1+z^{-1}}$, obtain transfer function of corresponding digital filter.

b) The pole of H(s) is s = -a and is in the left half plane of the complex plane. Find the pole of H(z) from $\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} = -a$ by assuming $aT \ll 1$ and aT > 1. In which case is it inside the unit circle?

Q5) The transfer function of a first order analog all pass filter is

$$H_a\left(s\right) = \frac{s-a}{s+a}$$

and the transfer function of a first order digital all pass filter is

$$H_d(z) = \frac{z^{-1} - a^*}{1 + az^{-1}}.$$

Compute $|H_a(j\omega)|$ and $|H_d(e^{j\Omega})|$.

Q6) We want to design a digital low pass filter having a pass-band $\Omega_0 = \frac{\pi}{4}$ rad/s by using bilinear s-to-z transformation method. The corresponding analog frequency of Ω_0 is

$$\omega_0 = \frac{2}{T} \tan\left(\frac{\Omega_0}{2}\right) = \frac{2}{T} \tan\left(\frac{\pi}{8}\right).$$

Using the first order analog low pass filter

$$H\left(s\right) = \frac{1}{1 + \frac{s}{w_0}}$$

obtain transfer function H(z) of the digital low pass filter. Note that $w_0T = 2\tan\left(\frac{\pi}{8}\right) = 2\left(\sqrt{2}-1\right)$.