Instructions Answer all questions. Give your answers clearly. Do not leave mathematical operations incomplete (do not skip intermediate operations and obtain the possible simplest form of the results). Laplace transform is bi-lateral if it is not stated otherwise. Calculator and cell phone are not allowed in the exam. Each question is worth 15 points. Time 120 minutes.


## QUESTIONS

Q1) Two continuous-time LTI systems, $\mathrm{S}_{\mathrm{I}}$ and $\mathrm{S}_{\text {II }}$ are given as in the following.

$$
\begin{aligned}
& \mathrm{S}_{\mathrm{I}}: y^{\prime}(t)+2 y(t)=x(t) \quad(\mathcal{D}+2) y(t)=x(t) \\
& \mathrm{S}_{\mathrm{II}}: y^{\prime}(t)+3 y(t)=x(t) \quad(\mathcal{D}+3) y(t)=x(t)
\end{aligned}
$$

a) The two systems are cascaded. Find the differential equation describing the equivalent system.
b) The two systems are connected in parallel. Find the differential equation describing the equivalent system.

Q2) A system function of an LTI causal system has two poles and one zero. The poles are $s=-3+2 j$ and $s=-3-2 j$ and the zero is $s=-1$. And the area under the impulse response of the system is unity;

$$
\int_{-\infty}^{\infty} h(t) d t=1
$$

Find the system function and the corresponding differential equation describing the system.
Q3) Two discrete-time LTI systems, $\mathrm{S}_{\mathrm{I}}$ and $\mathrm{S}_{\text {II }}$ are given as in the following.

$$
\begin{aligned}
& \mathrm{S}_{\mathrm{I}}: y[n]-\frac{1}{2} y[n-1]=x[n] \\
& \mathrm{S}_{\mathrm{II}}: y[n]-\frac{1}{3} y[n-1]=x[n] \\
& \left(1-\frac{1}{2} \mathcal{D}^{-1}\right) y[n]=x[n] \\
& \left(1-\frac{1}{3} \mathcal{D}^{-1}\right) y[n]=x[n]
\end{aligned}
$$

a) The two systems are cascaded. Find the difference equation describing the equivalent system.
b) The two systems are connected in parallel. Find the difference equation describing the equivalent system.

Q4) The following integral;

$$
\int_{-\infty}^{\infty} \delta^{\prime}(t) x(t) d t
$$

is equal to

$$
y(\lambda)=\delta^{\prime}(\lambda) * x(-\lambda)=\int_{-\infty}^{\infty} \delta^{\prime}(t) x(t-\lambda) d t
$$

at $\lambda=0$. We know that

$$
\begin{array}{ll}
\delta^{\prime}(t) * x(t) & =x^{\prime}(t) \\
{[x(-t)]^{\prime}} & =x(t)^{\prime}(-t)^{\prime}=-x^{\prime}(t)
\end{array}
$$


(Q1a, Q3a)

(Q1b, Q3b)

(Q2)

Find this integral.
Q5) An LTI-causal system is given as in the following

$$
y^{\prime \prime}(t)+4 y^{\prime}(t)+4 y(t)=x^{\prime}(t)+3 x(t)
$$

a) Find the system function. b) Compute the inverse Laplace transform of the system function to obtain the impulse response. If needed, use the Laplace transform property, $\mathcal{L}[\operatorname{tg}(t)]=-\frac{d}{d s} G(s)$.

Q6) Compute bi-lateral Laplace transform of the following signal by using the Laplace transform integral. Determine and plot region of convergence of the Laplace transform.

$$
x(t)=3 e^{-t} \cos (2 t) u(t)
$$

Q7) From the differentiation property of the uni-lateral Laplace transform we can write

$$
\mathcal{U} \mathcal{L}\left[x^{\prime}(t)\right]=s X(s)-x\left(0^{+}\right)=\int_{0^{+}}^{\infty} x^{\prime}(t) e^{-s t} d t
$$

Obtain the initial and final value theorems from the derivative property. Note that the integral variable $t$ is always positive.

Q8) A continuous-time signal; $x(t)$ is given as in the following figure. Find and plot $y(t)=\frac{d}{d t} x\left(-\frac{t}{2}-3\right)$.
You need to employ the chain rule; $\frac{d}{d t} g(a(t))=\frac{d}{d a} g(a) \frac{d}{d t} a(t)$, to obtain $y(t)$.
Q9) A continuous-time periodic signal is given as in the following figure. The function form of the signal is presented below.

$$
x(t)=\left\{\begin{array}{cc}
1+\cos (\pi t), & -1<t<0 \\
-1-\cos (\pi t), & 0<t<1 \\
x(t+2 \cdot \ell), & \ell \in \mathbb{Z}
\end{array}\right.
$$

Determine trigonometric Fourier series coefficients of the signal.
Q10) A discrete-time signal, $x[n]$, is given as in the following figure. Find and plot

$$
y[n]=x[-2 n+6]+x[2 n+1]+x[2 n-6]
$$


(Q8)

(Q9)

(Q10)

